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Probability Model for Estimating Three-Phase Relative Permeability

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Introduction

Although thorough analysis of combination gas- and water-drive reservoirs requires three-phase relative permeability data, the effort involved in determining these data experimentally generally rules out such a direct approach. Refs. 1 through 4 suggest, however, that more easily measured two-phase data can be used to predict the relative permeability to both the wetting and nonwetting fluids in three-phase flow. This report describes a method of using two sets of two-phase data to predict the relative permeability of the intermediate wettability phase in a three-phase system. Use is made of probability concepts and appropriate empirical definitions.

This technique may be regarded as a means of interpolating between the two sets of two-phase data to obtain the three-phase relative permeability. In many reservoirs that involve three-phase flow, only gas and oil are mobile in the upper portion of the reservoir; in the lower portion, water and oil are the phases of high mobility. The probability model is such that it will yield the correct two-phase data when only two phases are flowing, and will provide interpolated data for three-phase flow that are consistent and continuous functions of the phase saturations. It will be shown later that these interpolated values agree with the available three-phase data within experimental uncertainty.

Although the method applies to either a preferentially water-wet or a preferentially oil-wet system, this discussion will be limited to a water-wet system. Extension of the method to a preferentially oil-wet system is straightforward; here, water becomes the fluid of intermediate wettability.

Estimation of Three-Phase Relative Permeability Data

This section presents the data required to predict three-phase relative permeability data, the equations used, and the definitions and assumptions on which the method is based. The next section describes a reasonable physical model that is consistent with these assumptions, and the final section presents an empirical evaluation of the model.

Data Required

Data required for the estimation of three-phase relative permeability are two sets of two-phase data — water-oil and gas-oil. From the water-oil data we obtain both k_{rw} and k_{row} as a function of water saturation, where k_{row} is defined as the relative permeability to oil in the oil-water two-phase system.† Similarly, we obtain k_{rg} and k_{rog} as a function of gas saturation. Hysteresis effects are taken into consideration, as far as possible, by employing the appropriate two-phase data. For example, consider a water-wet system in which oil saturation is decreasing and gas and water saturations are increasing. Imbibition data should be used for the water-oil data, and drainage data should be used for the oil-gas data. However, if the water saturation is decreasing, then drainage data should

†We reserve the symbol k_{ro} for the oil relative permeability in the three-phase system.

With the method described here, three-phase relative permeability data may be estimated from two sets of more easily measured two-phase data — water displacing oil, and gas displacing oil. The resulting data compare favorably with the limited experimental data available in the literature, so that they may be used to estimate three-phase data for combination-drive reservoir calculations.

also be used ' · the water-oil system. It is not generally feasible to treat complicated hysteresis effects caused by oscillating saturations.

Equations

Refs. 1 through 4 suggest that for water-wet systems relative permeability to the wetting phase (water) is a function of the water saturation alone, and relative permeability to the nonwetting phase (gas) is a function of the gas saturation alone. Relative permeability of the fluid of intermediate wettability varies in a more complex manner, but the probability model uses the same two-phase data to predict it.

Normalized fluid saturations, given by Eqs. 1 through 3, are defined by treating connate water and irreducible residual oil as immobile fluids.

$$S_o^* = \frac{S_o - S_{om}}{(1 - S_{wc} - S_{om})} \text{ (for } S_o \geq S_{om}); \quad . \quad (1)$$

$$S_w^* = \frac{S_w - S_{wc}}{(1 - S_{wc} - S_{om})} \text{ (for } S_w \geq S_{wc}); \quad . \quad (2)$$

and

$$S_g^* = \frac{S_g}{(1 - S_{wc} - S_{om})} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

Note that $S_g^* + S_w^* + S_o^* = 1$.

When the normalized oil saturation S_o^* is 100 percent, k_{ro} also is 100 percent, but decreasing S_o^* (by increasing either the water or gas saturation) causes a decrease in k_{ro} which is greater than the decrease in S_o^*. We define β_w as a factor by which S_o^* is multiplied to allow for this disproportionate decrease in k_{ro}, due to the presence of water, and we assume β_w to be a function of the water saturation only. A similar factor for gas, $\beta_g = \beta_g(S_g^*)$, is also defined. We assume impedence of oil flow by water and gas to be mutually independent events, so that Eq. 4 can be written to relate the oil relative permeability to the oil saturation.

$$k_{ro} = S_o^* \beta_w \beta_g \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

Values of β_w as a function of water saturation are obtained from experimental k_{row} data by setting $\beta_g = 1$ and $S_g^* = 0$ in Eq. 4. When the resulting equation is solved for β_w, Eq. 5 results:

$$\beta_w = \frac{k_{row}}{1 - S_w^*} \quad \text{(two-phase data)} \quad . \; . \quad (5)$$

In Eq. 5, k_{row} is assumed to be a function of water saturation only, as determined by the two-phase experiment. Similarly, β_g can be developed as a function of S_g^* from experimental k_{rog} data by the relation

$$\beta_g = \frac{k_{rog}}{1 - S_g^*} \quad \text{(two-phase data)} \quad . \; . \quad (6)$$

Eqs. 1 through 6 define the relative permeability to oil in the three-phase system.

A special case arises for the relatively unimportant region in which $S_w \leq S_{wc}$. For this situation, k_{ro} is assumed to be a unique function of gas saturation regardless of the water saturation. Stated in terms of its physical implications, the water saturation deficiency ($S_{wc} - S_w$) must be made up by an equivalent oil saturation that will behave like the connate water and so will be immobile. To the extent that this assumption is correct, gas-oil relative permeability data measured in the absence of connate water can be used to determine β_g in Eq. 6.

Physical Model — Microscopic Fluid Distribution

The well known and widely accepted channel flow theory provides a physical basis for the assumptions made in developing the method for predicting three-phase relative permeability.

The Channel Flow Theory

The channel flow theory states that in any flow channel there is at most only one mobile fluid. A corollary of this theory is that the wetting phase is located primarily in the small pore spaces and the nonwetting phase in the large pore spaces, and the intermediate phase spatially separates them. It follows that at equal water saturations, the miscroscopic fluid distributions at the water-oil interface will be identical in a water-oil system and in a water-oil-gas system, so long as the direction of change of water saturation is the same in both. This implies that water relative permeability and water-oil capillary pressure in the three-phase system are functions of water saturations alone, irrespective of the relative saturations of oil and gas. Further, they are the same function in the three-phase system as in the two-phase water-oil system. Similarly, the gas phase relative permeability and gas-oil capillary pressure are the same functions of gas saturation in the three-phase system as in the two-phase gas-oil system. Such dependencies have been reported by previous investigators.[1-4]

Basis of Assumptions

The notion of identical microscopic fluid distributions around a two-phase interface is the basis for the assumptions that β_w and β_g are unique functions of water and gas saturations, respectively. The notion that water and gas are spatially remote leads us to the assumption that impedence of oil flow by water and gas are independent events.

One other assumption remains to be discussed — the treatment of connate water and an irreducible oil saturation as if they were immobile fluids. This is not a new concept when applied to connate water. However, if only the connate water is considered in normalizing the saturations, the model would predict that no residual oil is left either by gas displacement with connate water present, or by water displacement with sufficient gas present. While it is generally accepted that these residual oil saturations are small, it is not accepted that they are exactly zero. Unpublished data of Randall[5] show that residual oil remaining when gas displaces oil with connate water present may be of the order of one-fourth to one-half the connate water saturation. For practical application, it is desirable to specify a minimum value of the residual oil saturation, S_{om}, and this has been allowed for in Eqs. 1 through 3. In these equations, a constant value of S_{om} has been used. S_{om} is probably a function of the fluid saturations, but since sufficient data are not available to establish this dependence, S_{om} is treated as a constant.

Relation of Method to Probability Concepts

For an idealized porous system, Eq. 4 can be given a probabilistic interpretation. Let the porous system be a bundle of nearly identical capillaries of variable cross-section. Let each capillary through which oil is flowing be predominantly filled with oil, but consider that this flow may be blocked by the presence of a small amount of gas at one large cross-section, or water at one small cross-section. This water or gas comes from crossflow from capillaries filled with water or gas. While all of the oil-filled capillaries are nearly identical, they do have a random distribution of cross-section sizes at potential points of blockage, so that each channel will be blocked by water at a different average water saturation of the entire system.

If it were not for blocking by gas or water, then k_{ro} would equal S_o for the capillary system. To allow for blockage, the factor β_w is defined to equal the probability that an oil-filled capillary is not blocked by water, and β_g is similarly defined for gas. Since blockage by water occurs in one set of spatial points and blockage by gas in another, the two events are independent, and the probability of blockage by either water, gas, or both fluids would be the product $\beta_w \beta_g$, and k_{ro} would be given by Eq. 4.

In a more typical porous system, the flow channels are multiply cross-linked and blockage does not completely halt flow, but merely impedes it by diverting fluid around the blocked point. In such a system, β_w is given by Eq. 7, and β_g by a similar equation.

$$\beta_w = (\alpha_w f) + (1 - \alpha_w) \; , \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

where

α_w = probability of blockage by water at any one of many potential points of blockage

f = average fractional decrease in flow capacity of a channel due to a point's being blocked

The first term in Eq. 7 allows for the fraction of the points blocked by water and the decrease in the flow capacity due to this blockage. The second term is simply the fraction not blocked by water.

Although it is reasonable to expect α_w to be a unique function of water saturation, the factor f undoubtedly is a function of both water and gas saturation because of the possibility of channel blockage at a number of points. However, the accuracy of the available experimental data does not appear to warrant this refinement, and it has been neglected in this paper in which β_w has been assumed to be a function of water saturation only.

Test of Probability Model Against Experimental Data

Three sets of three-phase relative permeability data were used in checking the method. These data were taken from the work of Corey *et al.*,[1] Dalton *et al.*,[2] and Saraf.[3] Leverett and Lewis[4] also reported three-phase data, but since they were obtained without consideration of hysteresis effects, they exhibit too much experimental scatter to be useful for our purposes. Similarly, Snell's three-phase data[6] cannot be used because no corresponding two-phase data were provided. Calculated and measured k_{ro} values for the data of Corey, Dalton and Saraf are compared below.

Data of Corey *et al.*

Corey *et al.*[1] presented three-phase relative permeability data and data for k_{rog} as a function of liquid saturation for a Berea core. While no k_{row} data for the water-oil system were given, several values of k_{ro} were reported for small gas saturations. It was therefore possible to obtain the necessary k_{row} data by extrapolation to the water-oil base line on the ternary diagram presented in Corey's paper. The resulting k_{row} data and the reported k_{rog} data are shown in Fig. 1. Since the experimental procedure employed by Corey *et al.* involved oil displacing water, both the curves on Fig. 1 are drainage curves.

The data of Fig. 1 were used to predict three-phase k_{ro} data. Figs. 2 and 3 compare the experimental and the predicted values. These figures show lines of constant relative permeability (isoperms) on ternary diagrams for the oil-gas-water system. The solid curves correspond to $S_{om} = 0$; the dashed curves to $S_{om} = 0.1 = \frac{1}{2} S_{wc}$. Agreement of the predicted values for $S_{om} = 0$ with the experimental data is quite good, especially for the higher values of oil permeability. Use of a value of $S_{om} = 0.1$ does not alter this good agreement for high permeabilities and does improve the agreement at low permeabilities.

Data of Dalton, *et al.*

Fig. 4 depicts the two-phase relative permeability data for the sand system used by Dalton, *et al.*,[2] an unconsolidated Miocene sand pack. The connate water saturation was 15 percent for this system.

Figs. 5 and 6 compare predicted and measured values for k_{ro} for this three-phase system. The solid

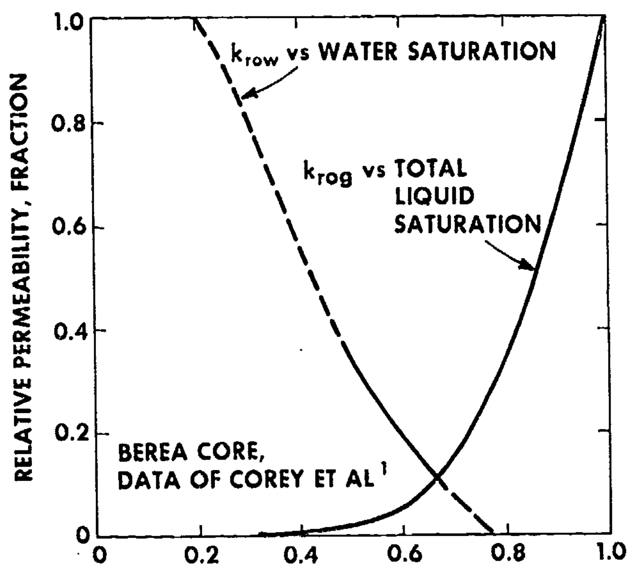

Fig. 1—Two-phase relative permeability data used in tests of probability model.

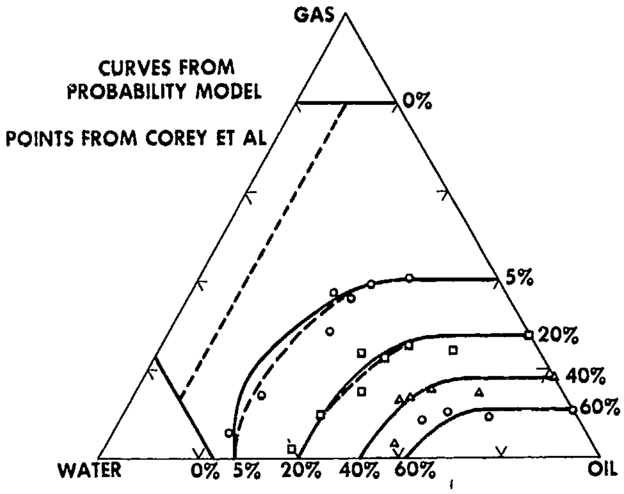

Fig. 2—Oil phase relative isoperms (Berea core).

TABLE 1—OIL RELATIVE PERMEABILITY IN OIL-WATER SYSTEM (k_{row}, SARAF)

Core No.	Water Saturation (percent)	Relative Permeability (percent)
6	30	56
11	35	40
7	40	38
4	41	34
2	49	20
9	52	19
1	55	17
10	60	11
5	75	4

TABLE 2—OIL RELATIVE PERMEABILITY IN GAS-OIL SYSTEM AT S_w = 25 PERCENT (k_{rog}, SARAF)

Gas saturation, percent	0	5	10	15	20	25	30	35
k_{rog}, percent	100	78.5	60.0	44.0	31.6	20.8	12.6	5.8

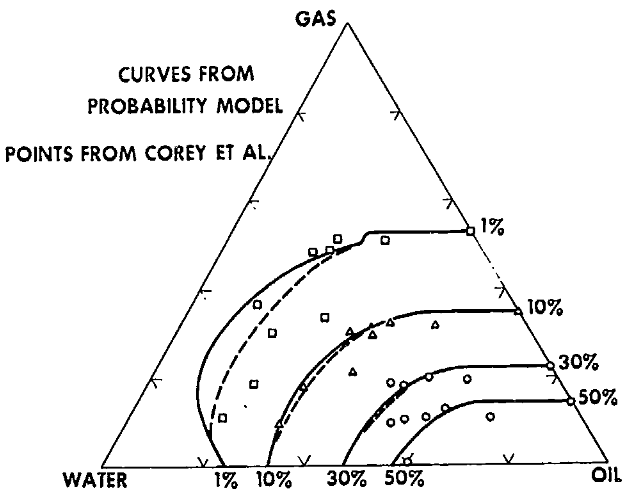

Fig. 3—Oil phase relative isoperms (Berea core).

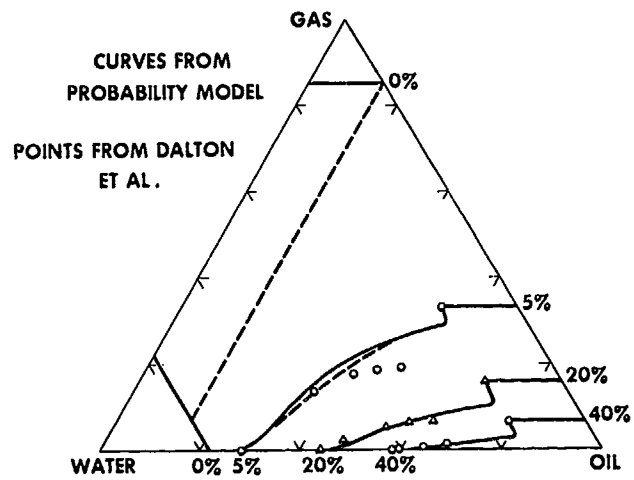

Fig. 5—Comparison of predicted and measured oil phase relative isoperms (unconsolidated Miocene sand, 10.8 darcies).

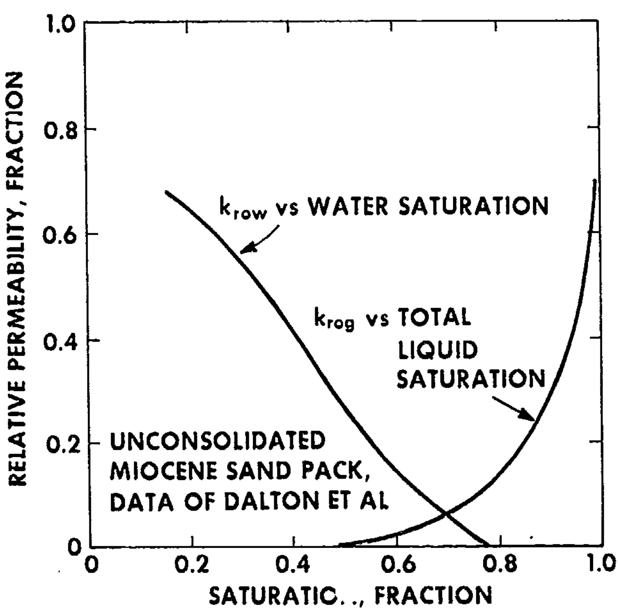

Fig. 4—Two-phase relative permeability data used in tests of probability model.

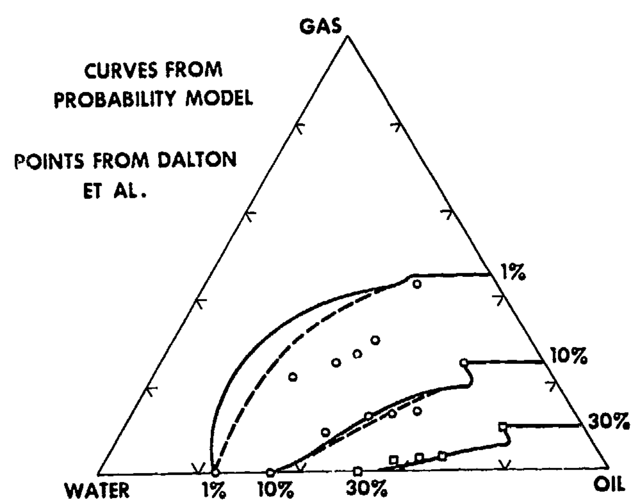

Fig. 6—Comparison of predicted and measured oil phase relative isoperms (unconsolidated Miocene sand, 10.8 darcies).

curve is for $S_{om} = 0$; the dashed one for $S_{om} = 0.15 = S_{wc}$. Agreement is quite good for relative permeability values of approximately 10 percent and greater. For lower relative permeability values, predicted and measured values for either values of S_{om} differ considerably.

Data of Saraf

Saraf[8] measured three-phase relative permeability data for Boise sandstone using Nuclear Magnetic Resonance to measure fluid saturations. A number of different, but similar, core samples were saturated with water, and then part of the water was displaced by oil. For each core, oil-water equilibrium was attained at a desired water saturation, and this saturation was held constant during all subsequent experiments with that core. Different water saturations were investigated by use of different core samples. The oil phase relative permeability in the two-phase oil-water system, k_{row}, was thus drainage data. The two-phase oil-water data points for each core, taken directly from Saraf's tabulated data, are given in Table 1.

Saraf did not obtain gas-oil data at the connate water saturation of 25 percent, but he did obtain one set of data, on Core 6, at a 30 percent water saturation. The gas-oil data that would best predict the 30 percent water saturation data are given in Table 2.

Using the data of Tables 1 and 2 and an S_{om} value of 20 percent, the maximum error in the relative permeability values predicted by the probability model for Core 6 was 6 percent, and the average error was only 1.5 percent.

Values of k_{ro} predicted for the other cores are given in Table 3, along with the experimentally observed data. As the table shows, agreement between predicted and experimental values is quite good. The maximum absolute difference in predicted and experimental percentage relative permeability values is 4.8. Even though relative errors are large for some low values of k_{ro}, the average relative deviation is only 17 percent. Considering only values of $k_{ro} \geq 5$ percent, this average is 12 percent.

TABLE 3—SARAF'S THREE-PHASE DATA

Run Number	Relative Permeability to Oil (percent)		Absolute Error (Predicted minus Experimental)	Relative Error (percent)
	Experimental	Predicted		
1c	12	14.2	2.2	18
1d	6	9.8	3.8	63
1e	< 1	4.2	>3.2	—
2f	16	17.1	1.1	7
2g	14	15.0	1.0	7
2h	10	12.0	2.0	20
2j	6.5	5.3	−1.2	−19
2k	0	0.9	—	—
4d	29	26.6	−2.3	− 8
4e	23.5	18.7	−4.8	−20
4f	19	14.8	−4.2	−22
4g	12	11.8	−0.2	− 2
4h	8	9.2	1.2	15
4j	3	6.9	3.6	120
4k	< 1	5.0	>4.0	—
4l	< 1	2.3	>1.3	—
4m	< 0.5	1.7	>1.2	—
4n	< 0.5	1.5	>1.0	—
4p	0	1.2	1.2	—
5d	3	2.0	−1.0	−33
5e	0	0.0	0	0
7c	31	30.0	−1.0	− 3
7d	24	24.0	0	0
7e	16	15.6	−0.4	− 3
7f	10	11.3	1.3	13
7g	< 1	0.3	—	—
7h	< 1	0.03	—	—
7i	0	0.0	—	—
9d	18	16.7	−1.3	− 7
9e	11	13.4	2.4	22
9f	5.5	6.0	0.5	9
9g	< 1	3.2	>2.2	—
9h	0	0.0	0	0
10d	7.5	6.3	1.2	16
10e	3	3.5	0.5	16
10f	< 1	2.6	>1.6	—
11f	40	39.0	−1.0	− 3
11g	35	35.2	0.2	1
11h	27	28.4	1.4	5
11i	23	24.8	1.8	8
11j	16.5	19.3	2.8	17
11k	14	13.1	−0.9	− 6
11l	10	10.4	0.4	4
11m	4.5	5.1	0.6	13
11n	2	2.9	0.9	45
11p	< 1	1.8	0.8	—
11q	< 1	0.8	—	—
11r	0	0.0	—	—

Nomenclature

All symbols, except those listed below, are SPE-AIME standard. Superscript * signifies a normalized saturation.

- f = average fractional decrease in flow capacity of a channel due to being blocked
- k_{row} = relative permeability to oil taken from water-oil data
- k_{rog} = relative permeability to oil taken from gas-oil data
- α_w = probability of channel blockage by water
- β_g = factor to allow for oil blockage by gas
- β_w = factor to allow for oil blockage by water
- S_{om} = minimum value of the residual oil saturation

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